Syllabus for Pre-Ph.D Entrance Test in Mathematics

DISCRETE STRUCTURES

I. Recurrence relations, Linear homogeneous recurrence relations, Non-homogeneous recurrence relations, Solutions of recurrence relations.

II. Partially ordered sets, Different type of lattices, Sub-lattices, Direct product, Ideal Lattice, Modular and distributive lattices.

III. Boolean algebra, Ideals in Boolean algebra, Boolean rings, Boolean functions, Karnaugh maps, Application of Boolean algebra to switching theory.

IV. Graphs, Direct graphs, Undirected graphs, Relations and graphs, Path and circuits, Eulerian and Hamiltonian graphs, Planner graphs, Connected graphs.

ABSTRACT ALGEBRA- I

I. Simple groups, Conjugacy, Normalization, Centre of a group, Class equation of a group and its consequences, Theorems for finite groups, Cauchy's theorem, Sylow's theorem.

II. Homomorphism, Endomorphism, Automorphism, Inner automorphism, Kernel of a homomorphism, Fundamental theorem on homomorphism of group, Group ofautomorphisms , Results on group homomorphism.

III. Maximal subgroups, Compositionseries, Jordan-Holder theorem, Solvable groups, Commutator subgroups, Direct products

IV. Ideals, Algebra of ideals, Principal ideal ring, Units and associates, Polynomials ring, Division and Euclidean algorithm for polynomials, Unique factorization theorem

MECHANICS

I. Conservation of linear and angular momentum under finite and impulsive forces, Conservation of energy.

II. Generalized coordinates, Lagrange's equations of motion, Small oscillations.

II. Hamiltonian's canonical equations, Hamilton's principle and principle of least action.

III. Euler's equations of motion, Kinetic energy, Eulerian angles, Instantaneous axis of rotation.

COMPLEX ANALYSIS

I. Power series of analytic functions,Convergence of power series,Radius of convergence,Taylor's and Laurent's series, Residue and poles, Singularities, Classification of singularities.

II. Residues,Residue at infinity,Cauchy residue theorem, Applications of residue theorem in evaluation of improper real integrals.

III. Conformal mapping: properties, Mobius transformation, Elementary examples.

IV. Maximum modulus theorem, Mittag-Leffler theorem,Rouche's theorem,Concept of entire functions with simple example, Analytic continuation.

OPERATIONS RESEARCH –I

I. An introduction to operations research, Methodology of O.R., Features of O.R. problems, Different models in O.R., Opportunities and shortcomings of O.R. approach.

II. Dual simplex method, Revised simplex method,Sensitivity analysis.

III. Assignment and Transportation problems.

IV. Theory of games, Integer linear programming.

ABSTRACT ALGEBRA-II

I. Embedding of rings, Ring of residue classes, Fundamental theorem on homomorphism of ring , Prime ideals, Maximal ideal.

II. Euclidean ring, Properties of Euclidean ring, Module, sub-module, Module homomorphism, Linear sum and direct sum of sub-module

III. Extension fields, Simple field extension, Algebraic field extension, Minimal polynomial, Roots of polynomials, Multiple roots, Splitting field.

IV. Automorphism of field, Fixed field, Normal extension, Galois group: Examples and characterizations, Construction with straight edge and compass.

OPERATIONS RESEARCH-II

I. Inventory control, Functional role of inventory control, Classification of EOQ models with shortages and without shortages.

II. Queuing theory, Characteristics of Queuing system, Probability distribution in queuing system, Single served queuing model, M|M|1 queuing models, Multiple server queuing models.

III. Markov chain, Application of Markov analysis, State and transition probabilities, Steady state conditions, Sequencing problems, Processing n jobs through two and three machines.

IV. Dynamic programming, Dynamic programming under certainty, Non-linear programming methods, Quadratic programming, Kuhn- Tucker conditions.

REAL ANALYSIS

I. The Riemann-Stieltjes Integral: Definition and existence of Riemann-Stieltjes integral, Properties of integrals, Integration and differentiation, Fundamental theorem of calculus, Integration of vector-valued functions.

II. Sequences and series of functions, Pointwise and uniform convergence, Cauchy criterion for uniform convergence, Uniform convergence and continuity, Uniform convergence and Riemann-Stieltjes integral, Uniform convergence and differentiation, Weierstrass approximation theorem.

III. Power series, Algebra of power series, Uniqueness theorem for power series, Abel's theorem, Taylor's theorem.

IV. Functions of several variables, Concept of functions of two variables, Continuity,Partial derivatives, Differentiability, Change of variables, The inverse function theorem, The implicit function theorem, Chain rule.

METRIC SPACES

I. Metric on a set, Pseudo-metrics, Equivalent metrics, Limit point, Closed sets, Adherent point, Dense subsets, Interior of a set and its properties, Subspaces, Product spaces.

II. Convergent sequences, Cauchy sequences, Algebra of convergent sequences, Subsequences, Continuity at a point, Continuity over a apace, Algebra of real valued continuous functions in a metric space, Homeomorphism, Isometries, Uniform continuity.

III. Complete metric spaces,Completeness and continuous mappings, Cantor's intersection theorem, Contraction mapping theorem, Connectedness in metric spaces, Properties of connectedness.

IV. Compact spaces, Compact subsets of the real line,Compactness and continuous mappings, Sequential compactness, Countable compactness, B-W property, B-W property and boundedness,B-W property and compactness, Compactness and uniform continuity, Lebesgue covering Lemma.

TOPOLOGY

I. Definition and examples of topological spaces, Closed sets, Closure, Dense subsets, Neighborhoods, Interior, Exterior and accumulation points, Bases and sub bases, subspaces, Product spaces and relative topology.

II. Continuous function, Homeomorphism, Connected and disconnected sets, Components, Locally connected spaces.

III. Countability axioms, First and second countable spaces, Lindelof's theorem, Separable spaces, Second countable and separability, Separable axioms: T_0,T_1 , T_2 ,T_3 , $T_{3\frac{1}{2}}$, T_4 and their characterizations.

IV. Compactness, Continuity and compact sets, Basic properties of compactness, Compactness and finite intersection property,Sequentially and countably compact sets, Local compactness, Tychonoff's theorem.

DIFFERENTIAL EQUATIONS

I. Ordinary differential equations: Qualitative properties of solution, Oscillation, Wronskian, Sturm separation and comparison theorem,Picard iteration methods, Uniqueness and existence theorem.

II. Ordinary points, Regular and singular points, Frobenius series solution for Legendre's and Bessel's differential equations with generating functions.

III. Classification of PDE of 2^{nd} order and canonical forms, Concept of separation of variable solution.

IV. Solution of heat diffusion,Laplace and wave equations,Non-linear partial differential equation of second order.

DIFFERENTIAL GEOMETRY

I. Curves in space; Arc length, Order of contact, Tangent, Normal, Binormal, Osculating, Plane, Serret-Frenet formulae, Curvature and torsion. Osculating circle and osculating sphere, Helix, Bertrand curves.
II. Behaviour of a curve in the neighbourhood of a point. Concept of a surface, Envelope and developable surface, Parametric curves, Family of the surfaces, Edge of regression, Ruled surfaces, Central points.
III. Fundamental forms and curvature of surfaces: First fundamental form. Second fundamental form of the surfaces of revolution, Weingarten's equation, Direction coefficients, Family of curves.
IV. Local non-intrinsic properties of a surface Normal curvature, Principal directions, Principal curvatures, Minimal surface, Lines of curvature. Rodrigues and Monge's theorem, Euler's theorem, Joachimisthal's theorem, Dupin's indicatrix, Third fundamental form.

MATHEMATICAL STATISTICS

I. Elements of probability, Sample space, Discrete probability, Baye's theorem, Random variables and distribution functions, Mathematical expectations and moments.
II. Some standard discrete and continuous univariate distributions: Binomial, Poisson, Normal, Gamma and Beta distributions.
III. Correlation, Rank correlation, Regression line, Multiple and partial correlation of three variables only, Data reduction techniques, Canonical correlation.
IV. Concepts of sampling, Stratified sampling and systematic sampling, Test of hypothesis: t,z ,chi square test.

CALCULUS OF VARIATIONS

I. Variation of functional,Continuity and differentiability of functional, Necessary condition for an extremum,Euler's equation,Variational problems in parametric form, Functional depending on higher order derivatives and variational problems with subsidiary condition.
II. The isoperimetric problem, Invariance of Euler's equation under coordinate transformation, General variational of functional, Variable end point problems,Transversality condition transversal theorem, Weierstrass-Endmann corner condition.
III. Sufficient condition for extremum: second variation,Legendre's and Jacobi's necessary condition, Canonical transformation, Noether's theorem, The principle of least action, Conservation law, Hamilton Jacobi's equations.

IV. Transformation of ODE and PDE into functionals and their solutions by Ritze, Galerkin, Collocation and Kantrovitch methods.

MEASURE AND INTEGRATION

I. Lebesgue outer measure, Measure of open and closed sets,Borel sets, Measurable sets,Measure of cantor’s ternary set, Non-measurable sets.

II. Measurable functions, Algebra of measurable functions, Step functions, Characteristic function, Simple functions, Convergence in measure, Egoroff’s theorem, Riesz theorem.

III. Lebesgue Integral and their properties, General Lebesgue integrals, Lebesgue integrals for unbounded functions, Convergence theorems, Fatou Lemma.

IV. Functions of bounded variations, Absolutely continuity, Variation function, Jordan-decomposition theorem, Indefinite integral and its characterizations, Differentiation of an integral, Lebesgue differentiation theorem.

FUNCTIONAL ANALYSIS

I. Normed linear spaces, Banach spaces, Subspaces, Quotient Spaces, Equivalent, Norms.

II. Bounded linear Transformation/operators, Hahn- Banach theorem, Open mapping theorem, Closed graph theorem, Uniform boundedness principle.

III. Inner product spaces, Hilbert spaces, Orthogonality of vectors, Orthogonal complements and projection theorem, Riesz representation theorem, Orthonormal Sets.

IV. Operators on Hilbert Spaces, Self-adjoint, Normal and unitary operators, Orthogonal projection operators.

LINEAR INTEGRAL EQUATIONS

I. Classification of integral equations, Relation between differential and integral Equations, Fredholm integral equations,Fredholm equations of second kind with separable kernels,Eigen values and eigen functions

II. Volterra integral equations, Resolvent kernel of Volterra equation, Convolution type kernel, Integral equations with symmetric kernel.

III. Method of successive approximation for Fredholm and Volterra equations of the second kind.

IV. Classical Fredholm theory,Singular integral equations,Hilbert type integral equations,Integral equation with Green’s function type kernels.

INTEGRAL TRANSFORMS

I: Orthogonal set of functions, Fourier series, Fourier sine and cosine series,Half range expansions, Fourier integral Theorem, Fourier Transform and their Basic Properties.

II: Fourier Cosine Transform, Fourier Sine Transform, Transforms of Derivatives, Fourier Transforms of simple Functions, Fourier Transforms of Rational Functions, Convolution Integral, Parseval's Theorem for Cosine and Sine Transforms, Inversion Theorem, Solutionof Partial Differential Equations using Fourier Transforms, Solution of Laplace and Diffusion equations.

III: Laplace Transform: Definition, Transform of some elementary functions, rules of manipulation of Laplace Transform, Transform of Derivatives, Relation involving Integrals, The error function, Transform of Bessel functions, Periodic functions, Convolution of two functions.

IV: Inverse Laplace Transform and their Properties, First & Second Shifting Properties,Inverse Laplace Transforms of Derivative and Integrals, Tauberian Theorem, Solution of Initial value problems for linear equations with constant coefficients, Linear differential equations with variable coefficients.